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Certain unified fractional integrals and derivatives for a product of Aleph function and a general class of multivariable polynomials

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Abstract

Saigo and Maeda (Transform Methods and Special Functions, Varna, Bulgaria, pp. 386-400, 1996) introduced and investigated certain generalized fractional integral and derivative operators involving the Appell function F_3. Here we aim at presenting four unified fractional integral and derivative formulas of Saigo and Maeda type, which are involved in a product of $\aleph$-function and a general class of multivariable polynomials. The main results, being of general nature, are shown to be some unification and extension of many known formulas given, for example, by Saigo and Maeda (Transform Methods and Special Functions, Varna, Bulgaria, pp. 386-400, 1996), Saxena *et al*. (Kuwait J. Sci. Eng. 35(1A):1-20, 2008), Srivastava and Garg (Rev. Roum. Phys. 32:685-692, 1987), Srivastava *et al*. (J. Math. Anal. Appl.193:373-389, 1995) and so on. Our main results are also shown to be further specialized to yield a large number of known and (presumably) new formulas involving, for instance, Saigo fractional calculus operators, several special functions such as H-function, I-function, Mittag-Leffler function, generalized Wright hypergeometric function, generalized Bessel-Maitland function.

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1 Introduction, definitions, and preliminaries

Fractional calculus deals with the investigations of integrals and derivatives of arbitrary orders. A remarkably large number of works on the subject of fractional calculus have given interesting account of the theory and applications of fractional calculus operators in many different areas of mathematical analysis (see, for very recent works, [1–8]).

The fractional integral operators, especially, involving various special functions have found significant importance and applications in various fields of applied mathematics. Since last five decades, a number of researchers like Love [9], Srivastava and Saxena [10], Debnath and Bhatta [11], Saxena *et al.* [12–15], Saigo [16], Samko *et al.* [17], Miller and Ross [18], and Ram and Kumar [19] and so on have studied, in depth, certain properties, applications, and different extensions of various hypergeometric operators of fractional integration.

Throughout this paper, let $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Z}_0^-$, and $\mathbb{N}$ denote the sets of complex numbers, real numbers, positive real numbers, nonpositive integers and positive integers, respectively, and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$.

Let $\alpha, \alpha', \beta, \beta', \gamma \in \mathbb{C}$. Then the fractional integral operators $I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}$ and $I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\gamma}$ of a function $f(x)$ are defined, for $\Re(\gamma) > 0$, as follows (see Saigo and Maeda [20]):

$$\bigl(I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma} f\bigr)(x) = \frac{x^{-\alpha}}{\Gamma(\gamma)} \int_0^x (x-t)^{\gamma-1} t^{-\alpha'} F_3\bigl(\alpha, \alpha', \beta, \beta'; \gamma; 1 - t/x, 1 - x/t\bigr) f(t)\, dt \tag{1.1}$$

and

$$\bigl(I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\gamma} f\bigr)(x) = \frac{x^{-\alpha'}}{\Gamma(\gamma)} \int_x^\infty (t-x)^{\gamma-1} t^{-\alpha} F_3\bigl(\alpha, \alpha', \beta, \beta'; \gamma; 1 - x/t, 1 - t/x\bigr) f(t)\, dt, \tag{1.2}$$

where F_3 is one of the Appell series defined by (see, *e.g.*, [21, p.23, Eq. (4)])

$$F_3\bigl(a, a', b, b'; c; x, y\bigr) = \sum_{m,n=0}^{\infty} \frac{(a)_m (a')_n (b)_m (b')_n}{(c)_{m+n}} \frac{x^m}{m!} \frac{y^n}{n!} \quad \bigl(\max\bigl\{|x|, |y|\bigr\} < 1\bigr) \tag{1.3}$$

and $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$) by (see [22, p.2 and pp.4-6]):

$$\begin{aligned}(\lambda)_n :&= \begin{cases} 1 & (n = 0), \\ \lambda(\lambda+1)\cdots(\lambda+n-1) & (n \in \mathbb{N}) \end{cases} \\ &= \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} \quad \bigl(\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-\bigr).\end{aligned} \tag{1.4}$$

Here Γ denotes the familiar gamma function.

These operators reduce to the following simpler fractional integral operators (see [16]):

$$I_{0,x}^{\alpha,0,\beta,\beta',\gamma} f(x) = I_{0,x}^{\gamma,\alpha-\gamma,-\beta} f(x) \quad (\gamma \in \mathbb{C}) \tag{1.5}$$

and

$$I_{x,\infty}^{\alpha,0,\beta,\beta',\gamma} f(x) = I_{x,\infty}^{\gamma,\alpha-\gamma,-\beta} f(x) \quad (\gamma \in \mathbb{C}). \tag{1.6}$$

Let $\alpha, \alpha', \beta, \beta', \gamma \in \mathbb{C}$ with $\Re(\gamma) > 0$ and $x \in \mathbb{R}_+$. Then the generalized fractional differentiation operators involving the Appell function F_3 in the kernel are defined as follows (see [20]):

$$\bigl(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} f\bigr)(x) = \bigl(I_{0+}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\bigr)(x) \tag{1.7}$$

$$= \left(\frac{d}{dx}\right)^n \bigl(I_{0+}^{-\alpha',-\alpha,-\beta'+n,-\beta,-\gamma+n} f\bigr)(x) \quad \bigl(\Re(\gamma) > 0; n := \bigl[\Re(\gamma)\bigr] + 1\bigr) \tag{1.8}$$

$$\begin{aligned}&= \frac{1}{\Gamma(n-\gamma)} \left(\frac{d}{dx}\right)^n \bigl(x^{\alpha'}\bigr) \int_0^x (x-t)^{n-\gamma-1} t^{\alpha} \\ &\quad \times F_3\left(-\alpha', -\alpha, n-\beta', -\beta, n-\gamma; 1 - \frac{t}{x}, 1 - \frac{x}{t}\right) f(t)\, dt\end{aligned} \tag{1.9}$$

and

$$\begin{aligned}\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x) &= \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma}f\right)(x) \qquad (1.10)\\ &= \left(-\frac{d}{dx}\right)^n \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta+n,-\gamma+n}f\right)(x)\\ &\quad \left(\Re(\gamma) > 0; n = \left[\Re(\gamma)\right] + 1\right) \qquad (1.11)\\ &= \frac{1}{\Gamma(n-\gamma)}\left(-\frac{d}{dx}\right)^n \left(x^{\alpha}\right)\int_x^{\infty} (t-x)^{n-\gamma-1} t^{\alpha'}\\ &\quad \times F_3\left(-\alpha', -\alpha, -\beta', n-\beta, n-\gamma; 1-\frac{x}{t}, 1-\frac{t}{x}\right) f(t)\, dt. \qquad (1.12)\end{aligned}$$

These operators reduce to the Saigo derivative operators as follows (see [16, 20]):

$$\left(D_{0+}^{0,\alpha',\beta,\beta',\gamma}f\right)(x) = \left(D_{0+}^{\gamma,\alpha'-\gamma,\beta'-\gamma}f\right)(x) \quad \left(\Re(\gamma) > 0\right) \qquad (1.13)$$

and

$$\left(D_{-}^{0,\alpha',\beta,\beta',\gamma}f\right)(x) = \left(D_{-}^{\gamma,\alpha'-\gamma,\beta'-\gamma}f\right)(x) \quad \left(\Re(\gamma) > 0\right). \qquad (1.14)$$

Furthermore we also have (see [20, p.394, Eqs. (4.18) and (4.19)])

$$\begin{aligned}&I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} x^{\rho-1} = \Gamma\left[\begin{matrix}\rho, \rho+\gamma-\alpha-\alpha'-\beta, \rho+\beta'-\alpha'\\ \rho+\gamma-\alpha-\alpha', \rho+\gamma-\alpha'-\beta, \rho+\beta'\end{matrix}\right] x^{\rho-\alpha-\alpha'+\gamma-1}\\ &\quad\left(\Re(\gamma) > 0, \Re(\rho) > \max\left\{0, \Re\left(\alpha+\alpha'+\beta-\gamma\right), \Re\left(\alpha'-\beta'\right)\right\}\right) \qquad (1.15)\end{aligned}$$

and

$$\begin{aligned}&I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} x^{\rho-1} = \Gamma\left[\begin{matrix}1+\alpha+\alpha'-\gamma-\rho, 1+\alpha+\beta'-\gamma-\rho, 1-\beta-\rho\\ 1-\rho, 1+\alpha+\alpha'+\beta'-\gamma-\rho, 1+\alpha-\beta-\rho\end{matrix}\right] x^{\rho-\alpha-\alpha'+\gamma-1}\\ &\quad\left(\Re(\gamma) > 0, \Re(\rho) < 1 + \min\left\{\Re(-\beta), \Re\left(\alpha+\alpha'-\gamma\right), \Re\left(\alpha+\beta'-\gamma\right)\right\}\right), \qquad (1.16)\end{aligned}$$

where the notation $\Gamma[\cdots]$ represents the fraction of gamma functions, for example,

$$\Gamma\left[\begin{matrix}\alpha, \beta, \gamma\\ a, b, c\end{matrix}\right] = \frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}{\Gamma(a)\Gamma(b)\Gamma(c)}.$$

Saxena and Saigo [23] presented the generalized fractional integral and derivative formulas of the H-function involving Saigo-Maeda fractional calculus operators. Similarly, generalized fractional calculus formulas of the Aleph function associated with the Appell function F_3 is given by Saxena *et al.* [14, 15], and Ram and Kumar [19].

Following Saxena and Pogány [24, 25], we define the Aleph function in terms of the Mellin-Barnes type integrals as follows (see also [26–28]):

$$\aleph[z] = \aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[z \,\middle|\, \begin{matrix}(b_j, B_j)_{1,m}, \ldots, [\tau_j(b_j, B_j)]_{m+1,q_i}\\ (a_j, A_j)_{1,n}, \ldots, [\tau_j(a_j, A_j)]_{n+1,p_i}\end{matrix}\right] := \frac{1}{2\pi i}\int_L \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\xi) z^{-\xi}\, d\xi, \qquad (1.17)$$

where $i = \sqrt{-1}$ and

$$\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\xi) = \frac{\prod_{j=1}^{m} \Gamma(b_j + B_j\xi) \prod_{j=1}^{n} \Gamma(1 - a_j - A_j\xi)}{\sum_{i=1}^{r} \tau_i \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - B_{ji}\xi) \prod_{j=n+1}^{p_i} \Gamma(a_{ji} + A_{ji}\xi)}. \tag{1.18}$$

The integration path $L = L_{i\gamma\infty}$, $\gamma \in \Re$ extends from $\gamma - i\infty$ to $\gamma + i\infty$, and is such that the poles of $\Gamma(1-a_j-A_j\xi)$, $j = \overline{1,n}$ (the symbol $\overline{1,n}$ is used for $1, 2, \dots, n$) do not coincide with the poles of $\Gamma(b_j + B_j\xi)$, $j = \overline{1,m}$. The parameters $p_i, q_i \in \mathbb{N}_0$ satisfy $0 \le n \le p_i$, $1 \le m \le q_i$, and $\tau_i > 0$ for $i = \overline{1,r}$. The parameters $A_j, B_j, A_{ji}, B_{ji} > 0$ and $a_j, b_j, a_{ji}, b_{ji} \in \mathbb{C}$. An empty product in (1.18) is interpreted as unity. The existence conditions for the defining integral (1.17) are given below:

$$\varphi_l > 0, \quad \left|\arg(z)\right| < \frac{\pi}{2}\varphi_l \quad (l = \overline{1,r}) \tag{1.19}$$

and

$$\varphi_l \ge 0, \quad \left|\arg(z)\right| < \frac{\pi}{2}\varphi_l \quad \text{and} \quad \Re(\zeta_l) + 1 < 0, \tag{1.20}$$

where

$$\varphi_l := \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_l \left(\sum_{j=n+1}^{p_l} A_{jl} + \sum_{j=m+1}^{q_l} B_{jl} \right) \tag{1.21}$$

and

$$\zeta_l := \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_l \left(\sum_{j=m+1}^{q_l} b_{jl} - \sum_{j=n+1}^{p_l} a_{jl} \right) + \frac{1}{2}(p_l - q_l) \quad (l = \overline{1,r}). \tag{1.22}$$

Remark For $\tau_i = 1$, $i = \overline{1,n}$, in (1.17), we get the I-function defined as follows (see Saxena [29]):

$$\begin{aligned} I^{m,n}_{p_i,q_i;r}[z] = \aleph^{m,n}_{p_i,q_i,1;r}[z] &= \aleph^{m,n}_{p_i,q_i,1;r}\left[z \,\middle|\, \begin{matrix} (b_j,B_j)_{1,m}, \dots, (b_j,B_j)_{m+1,q_i} \\ (a_j,A_j)_{1,n}, \dots, (a_j,A_j)_{n+1,p_i} \end{matrix} \right] \\ &:= \frac{1}{2\pi i} \int_L \Omega^{m,n}_{p_i,q_i,1;r}(\xi) z^{-\xi}\, d\xi, \end{aligned} \tag{1.23}$$

where the kernel $\Omega^{m,n}_{p_i,q_i,1;r}(\xi)$ is given in (1.18). The existence conditions for the integral in (1.23) are the same as given in (1.19)-(1.22) with $\tau_i = 1$ and $i = \overline{1,r}$.

If we set $r = 1$, then (1.23) reduces to the familiar H-function as follows (see [30]):

$$\begin{aligned} H^{m,n}_{p,q}[z] = \aleph^{m,n}_{p_i,q_{i,1;1}}[z] &= \aleph^{m,n}_{p_i,q_{i,1;1}}\left[z \,\middle|\, \begin{matrix} (b_p,B_p) \\ (a_p,A_p) \end{matrix} \right] \\ &:= \frac{1}{2\pi i} \int_L \Omega^{m,n}_{p_i,q_i,1;1}(\xi) z^{-\xi}\, d\xi, \end{aligned} \tag{1.24}$$

where the kernel $\Omega^{m,n}_{p_i,q_i,1;1}(\xi)$ can be obtained from (1.18).

A general class of multivariable polynomials is defined and studied by Srivastava and Garg [31]:

$$S_L^{h_1,\ldots,h_s} = \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s)\frac{x_1^{k_1}}{k_1!}\cdots\frac{x_s^{k_s}}{k_s!}$$
$$(h_i\in\mathbb{N}, i=1,\ldots,s), \tag{1.25}$$

where $h_1,\ldots,h_s$ are arbitrary positive integers and the coefficients $A(L;k_1,\ldots,k_s)$, $(L,h_i\in\mathbb{N}_0; i=1,\ldots,s)$ are arbitrary constants, real or complex. Evidently the case $s=1$ of the polynomials (1.24) would correspond to the polynomials due to Srivastava [32] as

$$S_l^h(x) = \sum_{k=0}^{[l/h]} \frac{(-l)_{hk}}{k!}A_{l,k}x^k \quad (l\in\mathbb{N}_0).$$

Some multidimensional fractional integral operators involving the polynomial given as (1.25) are defined and studied by Srivastava *et al.* [33].

Here, in this paper, we aim at presenting four unified fractional integral and derivative formulas of Saigo and Maeda type [20], which are involved in a product of $\aleph$-function (1.17) and a general class of multivariable polynomials (1.25). The main results, being of general nature, are shown to be some unification and extension of many known formulas given, for example, by Saigo and Maeda [20], Saxena *et al.* [13], Srivastava and Garg [31], Srivastava *et al.* [33] and so on. Our main results are also shown to be further specialized to yield a large number of known and (presumably) new formulas involving, for instance, Saigo fractional calculus operators, several special functions such as H-function, I-function, Mittag-Leffler function, generalized Wright hypergeometric function, generalized Bessel-Maitland function.

2 Fractional integral formulas

Here we establish two fractional integration formulas for $\aleph$-function (1.17) and a general class of polynomials defined by (1.25).

Theorem 1 *Suppose that* $\alpha,\alpha',\beta,\beta',\gamma,z,\rho\in\mathbb{C}$, $\Re(\gamma)>0$, $\mu>0$, $\lambda_j\in\Re_+$ $(j=1,\ldots,s)$, *and*

$$\Re(\rho)+\mu\min_{1\leq j\leq m}\frac{\Re(b_j)}{B_j} > \max\{0,\Re(\alpha+\alpha'+\beta-\gamma),\Re(\alpha'-\beta')\}.$$

Further suppose that the constants $a_j,b_j,a_{ji},b_{ji}\in\mathbb{C}$, $A_j,B_j,A_{ji},B_{ji}\in\mathbb{R}_+$ $(i=1,\ldots,p_i;\ j=1,\ldots,q_i)$, *and* $\tau_i>0$ *for* $i=\overline{1,r}$. *If the conditions given in* (1.19)-(1.22) *are satisfied, then the following relation holds true*:

$$\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_L^{h_1,\ldots,h_s}(y_1t^{\lambda_1},\ldots,y_st^{\lambda_s})\right.\right.$$
$$\left.\left.\times\aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[zt^{\mu}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{matrix}\right]\right)\right\}(x)$$
$$= x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L}(-L)_{h_1k_1+\cdots+h_sk_s}A(L;k_1,\ldots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{\sum_{j=1}^s\lambda_jk_j}$$

$$\times \aleph^{m,n+3}_{p_i+3,q_i+3,\tau_i;r}\left[zx^{\mu}\,\middle|\,\begin{array}{l}(1-\rho-\sum_{j=1}^{s}\lambda_jk_j,\mu),(1-\rho+\alpha+\alpha'+\beta-\gamma-\sum_{j=1}^{s}\lambda_jk_j,\mu),\\ (b_j,B_j)_{1,m},(1-\rho+\alpha+\alpha'-\gamma-\sum_{j=1}^{s}\lambda_jk_j,\mu),\end{array}\right.$$
$$\left.\begin{array}{l}(1-\rho+\alpha'-\beta'-\sum_{j=1}^{s}\lambda_jk_j,\mu),(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\ (1-\rho+\alpha'+\beta-\gamma-\sum_{j=1}^{s}\lambda_jk_j,\mu),(1-\rho-\beta'-\sum_{j=1}^{s}\lambda_jk_j,\mu),\ldots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{array}\right]. \tag{2.1}$$

Proof In order to prove (2.1), first expressing the general class of multivariable polynomials occurring on its left-hand side as the series given by (1.25), replacing the $\aleph$-function in terms of Mellin-Barnes contour integral with the help of (1.17), interchanging the order of summations, we obtain the following form (say I):

$$\begin{aligned}I &= \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L}(-L)_{h_1k_1+\cdots+h_sk_s}A(L;k_1,\ldots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}\\ &\quad\times\left\{\frac{1}{2\pi i}\int_L\Omega^{m,n}_{p_i,q_i,\tau_i;r}z^{-\xi}\left(I^{\alpha,\alpha',\beta,\beta',\gamma}_{0+}t^{\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi-1}\right)(x)\,d\xi\right\}\\ &= \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L}(-L)_{h_1k_1+\cdots+h_sk_s}A(L;k_1,\ldots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}\\ &\quad\times\frac{1}{2\pi i}\int_L x^{\rho-\alpha-\alpha'+\gamma+\sum_{j=1}^{s}\lambda_jk_j-1}\left(zx^{\mu}\right)^{-\xi}\\ &\quad\times\frac{\prod_{j=1}^{m}\Gamma(b_j+B_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j-A_j\xi)}{\sum_{i=1}^{r}\tau_i\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}\xi)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}\xi)}\\ &\quad\times\frac{\Gamma(\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi)}{\Gamma(\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi+\gamma-\alpha-\alpha')}\\ &\quad\times\frac{\Gamma(\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi+\gamma-\alpha-\alpha'-\beta)\Gamma(\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi+\beta'-\alpha')}{\Gamma(\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi+\gamma-\alpha'-\beta)\Gamma(\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi+\beta')}\,d\xi.\end{aligned}$$

Finally, re-interpreting the Mellin-Barnes contour integral in terms of the $\aleph$-function, we are led to the right-hand side of (2.1). This completes proof of Theorem 1. □

In view of the relation (1.5), we obtain a (presumably) new result concerning the Saigo fractional integral operator [16] asserted by the following corollary.

Corollary 1 *Let* $\alpha,\beta,\gamma,\rho,z\in\mathbb{C}$, $\Re(\alpha)>0$, $\mu>0$, $\lambda_j\in\Re_+$ $(j=1,\ldots,s)$, *and*

$$\Re(\rho)+\mu\min_{1\leq j\leq m}\left(\frac{\Re(b_j)}{B_j}\right)>\max\{0,\Re(\beta-\gamma)\}.$$

Then the following relation holds true:

$$\begin{aligned}&\left\{I^{\alpha,\beta,\gamma}_{0+}\left(t^{\rho-1}S_L^{h_1,\ldots,h_s}\left(y_1t^{\lambda_1},\ldots,y_st^{\lambda_s}\right)\right.\right.\\ &\quad\left.\left.\times\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[zt^{\mu}\,\middle|\,\begin{array}{l}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\ (b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{array}\right]\right)\right\}(x)\\ &= x^{\rho-\beta-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L}(-L)_{h_1k_1+\cdots+h_sk_s}A(L;k_1,\ldots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{\sum_{j=1}^{s}\lambda_jk_j}\end{aligned}$$

$$\times \aleph^{m,n+2}_{p_i+2,q_i+2,\tau_i;r}\left[zx^{\mu}\,\middle|\,\begin{matrix}(1-\rho-\sum_{j=1}^{s}\lambda_j k_j,\mu),(1-\rho+\beta-\gamma-\sum_{j=1}^{s}\lambda_j k_j,\mu),\\ (b_j,B_j)_{1,m},(1-\rho+\beta-\sum_{j=1}^{s}\lambda_j k_j,\mu),\end{matrix}\right.$$
$$\left.\begin{matrix}(a_j,A_j)_{1,n},\dots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\ (1-\rho-\alpha-\gamma-\sum_{j=1}^{s}\lambda_j k_j,\mu),\dots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{matrix}\right], \tag{2.2}$$

where the conditions of the existence of (2.2) *follow easily with the help of* (2.1).

It is remarked in passing that the corresponding results concerning Riemann-Liouville and Erdélyi-Kober fractional integral operators can be obtained by putting $\beta=-\alpha$ and $\beta=0$, respectively, in (2.2).

Theorem 2 *Suppose that* $\alpha,\alpha',\beta,\beta',\gamma,z,\rho\in\mathbb{C}$, $\Re(\gamma)>0$, $\mu>0$, $\lambda_j\in\Re_+$ $(j=1,\dots,s)$, *and*

$$\Re(\rho)+\mu\max_{1\le i\le n}\left(\frac{\Re(a_i)-1}{A_i}\right)<1+\min\bigl\{\Re(-\beta),\Re\bigl(\alpha+\alpha'-\gamma\bigr),\Re\bigl(\alpha+\beta'-\gamma\bigr)\bigr\}.$$

Further suppose that the constants $a_j,b_j,a_{ji},b_{ji}\in\mathbb{C}$,

$$A_j,B_j,A_{ji},B_{ji}\in\mathbb{R}_+\quad(i=1,\dots,p_i;j=1,\dots,q_i),$$

and $\tau_i>0$ *for* $i=\overline{1,r}$. *If the conditions given in* (1.19)-(1.22) *are satisfied, the following relation holds true*:

$$\Biggl\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\Biggl(t^{\rho-1}S_L^{h_1,\dots,h_s}\bigl(y_1t^{\lambda_1},\dots,y_st^{\lambda_s}\bigr)$$
$$\times\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[zt^{\mu}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n},\dots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\(b_j,B_j)_{1,m},\dots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{matrix}\right]\Biggr)\Biggr\}(x)$$
$$=x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}(-L)_{h_1k_1+\dots+h_sk_s}A(L;k_1,\dots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{\sum_{j=1}^{s}\lambda_jk_j}$$
$$\times\aleph^{m+3,n}_{p_i+3,q_i+3,\tau_i;r}\left[zx^{\mu}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n},(1-\rho-\sum_{j=1}^{s}\lambda_jk_j,\mu),(1+\alpha-\beta-\rho-\sum_{j=1}^{s}\lambda_jk_j,\mu),\\(1+\alpha+\alpha'-\gamma-\rho-\sum_{j=1}^{s}\lambda_jk_j,\mu),(1-\beta-\rho-\sum_{j=1}^{s}\lambda_jk_j,\mu),\end{matrix}\right.$$
$$\left.\begin{matrix}(1+\alpha+\alpha'+\beta'-\gamma-\rho-\sum_{j=1}^{s}\lambda_jk_j,\mu),\dots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\(1+\alpha+\beta'-\gamma-\rho-\sum_{j=1}^{s}\lambda_jk_j,\mu),(b_j,B_j),\dots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{matrix}\right]. \tag{2.3}$$

Proof A similar argument as in proving Theorem 1 will establish the result (2.3). Indeed, first expressing the general class of multivariable polynomials occurring on its left-hand side as a series given by (1.25), replacing the ℵ-function in terms of Mellin-Barnes contour integral with the help of (1.17), interchanging the order of summations, we obtain the following form (say I):

$$I=\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}(-L)_{h_1k_1+\dots+h_sk_s}A(L;k_1,\dots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}$$
$$\times\left\{\frac{1}{2\pi i}\int_L\Omega^{m,n}_{p_i,q_i,\tau_i;r}z^{-\xi}\bigl(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho+\sum_{j=1}^{s}\lambda_jk_j-\mu\xi-1}\bigr)(x)\,d\xi\right\}$$

$$= \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}$$
$$\times \frac{1}{2\pi i}\int_L x^{\rho-\alpha-\alpha'+\gamma+\sum_{j=1}^s \lambda_j k_j -1}\big(zx^\mu\big)^{-\xi}$$
$$\times \frac{\prod_{j=1}^m \Gamma(b_j+B_j\xi)\prod_{j=1}^n \Gamma(1-a_j-A_j\xi)}{\sum_{i=1}^r \tau_i \prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}\xi)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}\xi)}$$
$$\times \frac{\Gamma(1+\alpha+\alpha'-\gamma-\rho-\sum_{j=1}^s\lambda_jk_j+\mu\xi)}{\Gamma(1-\rho-\sum_{j=1}^s\lambda_jk_j+\mu\xi)}$$
$$\times \frac{\Gamma(1+\alpha+\beta'-\gamma-\rho-\sum_{j=1}^s\lambda_jk_j+\mu\xi)\Gamma(1-\beta-\rho-\sum_{j=1}^s\lambda_jk_j+\mu\xi)}{\Gamma(1+\alpha+\alpha'+\beta'-\gamma-\rho-\sum_{j=1}^s\lambda_jk_j+\mu\xi)\Gamma(1+\alpha-\beta-\rho-\sum_{j=1}^s\lambda_jk_j+\mu\xi)}\,d\xi.$$

Finally, re-interpreting the Mellin-Barnes contour integral in terms of the ℵ-function, we are led to the right-hand side of (2.3). This completes the proof of Theorem 2. □

In view of the relation (1.6), we obtain a (presumably) new result concerning Saigo fractional integral operator [16] asserted by the following corollary.

Corollary 2 Let $\alpha,\beta,\gamma,\rho,z\in\mathbb{C}$, $\Re(\alpha)>0$, $\mu>0$, $\lambda_j\in\Re_+$ $(j=1,\ldots,s)$, *and*

$$\Re(\rho)+\mu\max_{1\le i\le n}\left(\frac{\Re(a_i)-1}{A_i}\right)<1+\min\bigl\{\Re(\beta),\Re(\gamma)\bigr\}.$$

Then the following relation holds true:

$$\left\{I_-^{\alpha,\beta,\gamma}\left(t^{\rho-1}S_L^{h_1,\ldots,h_s}\big(y_1t^{\lambda_1},\ldots,y_st^{\lambda_s}\big)\right.\right.$$
$$\left.\left.\times\, \aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[zt^\mu \,\middle|\, \begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\ (b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{matrix}\right]\right)\right\}(x)$$
$$= x^{\rho-\beta-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L}(-L)_{h_1k_1+\cdots+h_sk_s}A(L;k_1,\ldots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{\sum_{j=1}^s\lambda_jk_j}$$
$$\times\,\aleph_{p_i+2,q_i+2,\tau_i;r}^{m+2,n}\left[zx^\mu\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n},(1-\rho-\sum_{j=1}^s\lambda_jk_j,\mu),\\ (1+\beta-\rho-\sum_{j=1}^s\lambda_jk_j,\mu),(1+\gamma-\rho-\sum_{j=1}^s\lambda_jk_j,\mu),\end{matrix}\right.$$
$$\left.\begin{matrix}(1+\alpha+\beta+\gamma-\rho-\sum_{j=1}^s\lambda_jk_j,\mu),\ldots,[\tau_j(a_j,A_j)]_{n+1,p_i}\\ (b_j,B_j),\ldots,[\tau_j(b_j,B_j)]_{m+1,q_i}\end{matrix}\right], \tag{2.4}$$

where the conditions of existence of (2.4) *follow easily from Theorem* 2.

It is also remarked in passing that the corresponding results concerning Riemann-Liouville and Erdélyi-Kober fractional integral operators can be obtained by putting $\beta=-\alpha$ and $\beta=0$, respectively, in (2.4).

3 Fractional derivative formulas

Here we establish two fractional derivative formulas for ℵ-function (1.17) and a general class of polynomials defined by (1.25).

Theorem 3 *Suppose that* $\alpha, \alpha', \beta, \beta', \gamma, z, \rho \in \mathbb{C}$, $\Re(\gamma) > 0$, $\mu > 0$, $\lambda_j \in \Re_+$ $(j = 1, \ldots, s)$, *and*

$$\Re(\rho) + \mu \min_{1 \le j \le m} \left(\frac{\Re(b_j)}{B_j} \right) + \max\left\{0, \Re(\alpha - \beta), \Re\left(\alpha' + \beta' + \alpha - \gamma\right)\right\} > 0.$$

Further suppose that the constants $a_j, b_j, a_{ji}, b_{ji} \in \mathbb{C}$, $A_j, B_j, A_{ji}, B_{ji} \in \mathbb{R}_+$ $(i = 1, \ldots, p_i; j = 1, \ldots, q_i)$, *and* $\tau_i > 0$ *for* $i = \overline{1, r}$. *If the conditions given in* (1.19)-(1.22) *are satisfied, then the following relation holds true*:

$$\begin{aligned}
&\Bigg\{ D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} \Bigg(t^{\rho-1} S_L^{h_1,\ldots,h_s}\left(y_1 t^{\lambda_1}, \ldots, y_s t^{\lambda_s}\right) \\
&\quad \times \aleph_{p_i,q_i,\tau_i;r}^{m,n} \left[z t^{\mu} \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, \ldots, [\tau_j(a_j, A_j)]_{n+1,p_i} \\ (b_j, B_j)_{1,m}, \ldots, [\tau_j(b_j, B_j)]_{m+1,q_i} \end{matrix} \right] \Bigg) \Bigg\}(x) \\
&= x^{\rho+\alpha+\alpha'-\gamma-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s \le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L; k_1, \ldots, k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j} \\
&\quad \times \aleph_{p_i+3,q_i+3,\tau_i;r}^{m,n+3} \left[z x^{\mu} \,\middle|\, \begin{matrix} (1-\rho-\sum_{j=1}^s \lambda_j k_j, \mu), (1-\rho-\alpha+\beta-\sum_{j=1}^s \lambda_j k_j, \mu), \\ (b_j, B_j)_{1,m}, (1-\rho-\alpha-\alpha'+\gamma-\sum_{j=1}^s \lambda_j k_j, \mu), \end{matrix} \right. \\
&\qquad \left. \begin{matrix} (1-\rho-\alpha-\alpha'-\beta'+\gamma-\sum_{j=1}^s \lambda_j k_j, \mu), (a_j, A_j)_{1,n}, \ldots, [\tau_j(a_j, A_j)]_{n+1,p_i} \\ (1-\rho-\alpha-\beta'+\gamma-\sum_{j=1}^s \lambda_j k_j, \mu), (1-\rho+\beta-\sum_{j=1}^s \lambda_j k_j, \mu), \ldots, [\tau_j(b_j, B_j)]_{m+1,q_i} \end{matrix} \right].
\end{aligned} \tag{3.1}$$

Proof In order to prove (3.1), first expressing the general class of multivariable polynomials occurring on its left-hand side as a series given by (1.25), replacing the $\aleph$-function in terms of the Mellin-Barnes contour integral with the help of (1.17), and interchanging the order of summations, we obtain the following form (say I):

$$\begin{aligned}
I &= \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s \le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L; k_1, \ldots, k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!} \\
&\quad \times \left\{ \frac{1}{2\pi i} \int_L \Omega_{p_i,q_i,\tau_i;r}^{m,n} z^{-\xi} \left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho+\sum_{j=1}^s \lambda_j k_j - \mu\xi - 1} \right)(x)\, d\xi \right\} \\
&= \left(\frac{d}{dx} \right)^n \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s \le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L; k_1, \ldots, k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!} \\
&\quad \times \frac{1}{2\pi i} \int_L x^{\rho+\alpha+\alpha'-\gamma+\sum_{j=1}^s \lambda_j k_j + n - 1} \left(z x^{\mu} \right)^{-\xi} \\
&\quad \times \frac{\prod_{j=1}^m \Gamma(b_j + B_j\xi) \prod_{j=1}^n \Gamma(1 - a_j - A_j\xi)}{\sum_{i=1}^r \tau_i \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - B_{ji}\xi) \prod_{j=n+1}^{p_i} \Gamma(a_{ji} + A_{ji}\xi)} \\
&\quad \times \frac{\Gamma(\rho + \sum_{j=1}^s \lambda_j k_j - \mu\xi)}{\Gamma(\rho + \sum_{j=1}^s \lambda_j k_j - \mu\xi + \alpha + \alpha' - \gamma)} \\
&\quad \times \frac{\Gamma(\rho + \sum_{j=1}^s \lambda_j k_j - \mu\xi + \alpha + \alpha' + \beta' - \gamma) \Gamma(\rho + \sum_{j=1}^s \lambda_j k_j - \mu\xi + \alpha - \beta)}{\Gamma(\rho + \sum_{j=1}^s \lambda_j k_j - \mu\xi + \alpha + \beta' - \gamma) \Gamma(\rho + \sum_{j=1}^s \lambda_j k_j - \mu\xi - \beta)}\, d\xi.
\end{aligned}$$

Here $n := [-\Re(\gamma)] + 1$, and by using

$$\frac{d^k}{dx^k} x^m = \frac{\Gamma(m+1)}{\Gamma(m-k+1)} x^{m-k} \quad (m, k \in \mathbb{N}_0; m \ge k), \tag{3.2}$$

and re-interpreting the Mellin-Barnes counter integral in terms of the $\aleph$-function, we are led to the right-hand side of (3.1). This completes the proof of Theorem 3. $\square$

In view of the relation (1.13), we obtain a (presumably) new result concerning Saigo fractional derivative operator [16] asserted by the following corollary.

Corollary 3 *Let* $\alpha, \beta, \gamma, \rho, z \in \mathbb{C}$, $\Re(\alpha) > 0$, $\mu > 0$, $\lambda_j \in \Re_+$ $(j = 1, \ldots, s)$, *and*

$$\Re(\rho) + \mu \min_{1 \le j \le m} \left(\frac{\Re(b_j)}{B_j} \right) + \max\bigl\{0, \Re(\beta), \Re(\beta + \alpha + \gamma)\bigr\} > 0.$$

Then the following relation holds true:

$$\begin{aligned}
&\Biggl\{ D_{0+}^{\alpha,\beta,\gamma} \Biggl(t^{\rho-1} S_L^{h_1,\ldots,h_s}\bigl(y_1 t^{\lambda_1}, \ldots, y_s t^{\lambda_s}\bigr) \\
&\quad \times \aleph_{p_i,q_i,\tau_i;r}^{m,n} \left[z t^{\mu} \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, \ldots, [\tau_j(a_j, A_j)]_{n+1,p_i} \\ (b_j, B_j)_{1,m}, \ldots, [\tau_j(b_j, B_j)]_{m+1,q_i} \end{matrix} \right] \Biggr) \Biggr\}(x) \\
&= x^{\rho+\beta-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s \le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L; k_1, \ldots, k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j} \\
&\quad \times \aleph_{p_i+2,q_i+2,\tau_i;r}^{m,n+2} \left[z x^{\mu} \,\middle|\, \begin{matrix} (1-\rho-\sum_{j=1}^s \lambda_j k_j, \mu), (1-\rho-\alpha-\beta-\gamma-\sum_{j=1}^s \lambda_j k_j, \mu), \\ (b_j, B_j)_{1,m}, (1-\rho-\beta-\sum_{j=1}^s \lambda_j k_j, \mu), \end{matrix} \right. \\
&\qquad \left. \begin{matrix} (a_j, A_j)_{1,n}, \ldots, [\tau_j(a_j, A_j)]_{n+1,p_i} \\ (1-\rho-\gamma-\sum_{j=1}^s \lambda_j k_j, \mu), \ldots, [\tau_j(b_j, B_j)]_{m+1,q_i} \end{matrix} \right],
\end{aligned} \tag{3.3}$$

where the conditions of existence of (3.3) *follow easily with the help of* (3.1).

It is remarked in passing that the corresponding results concerning Riemann-Liouville and Erdélyi-Kober fractional integral operators can be obtained by putting $\beta = -\alpha$ and $\beta = 0$, respectively, in (3.3).

Theorem 4 *Suppose that* $\alpha, \alpha', \beta, \beta', \gamma, z, \rho \in \mathbb{C}$, $\Re(\gamma) > 0$, $\mu > 0$, $\lambda_j \in \Re_+$ $(j = 1, \ldots, s)$, *and*

$$\Re(\rho) + \mu \max_{1 \le i \le n} \left(\frac{\Re(a_i) - 1}{A_i} \right) < 1 + \min\bigl\{\Re(-\beta), \Re\bigl(\gamma - \alpha - \alpha' - k\bigr), \Re\bigl(-\alpha' - \beta + \gamma\bigr)\bigr\},$$

here $k = [\Re(\gamma)] + 1$. *Further suppose that the constants* $a_j, b_j, a_{ji}, b_{ji} \in \mathbb{C}$,

$$A_j, B_j, A_{ji}, B_{ji} \in \mathbb{R}_+ \quad (i = 1, \ldots, p_i; j = 1, \ldots, q_i),$$

and $\tau_i > 0$ *for* $i = \overline{1, r}$. *If the conditions given in* (1.19)-(1.22) *are satisfied, the following relation holds true*:

$$\begin{aligned}
&\Biggl\{ D_{-}^{\alpha,\alpha',\beta,\beta',\gamma} \Biggl(t^{\rho-1} S_L^{h_1,\ldots,h_s}\bigl(y_1 t^{\lambda_1}, \ldots, y_s t^{\lambda_s}\bigr) \\
&\quad \times \aleph_{p_i,q_i,\tau_i;r}^{m,n} \left[z t^{\mu} \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, \ldots, [\tau_j(a_j, A_j)]_{n+1,p_i} \\ (b_j, B_j)_{1,m}, \ldots, [\tau_j(b_j, B_j)]_{m+1,q_i} \end{matrix} \right] \Biggr) \Biggr\}(x)
\end{aligned}$$

$$= (-1)^{[\Re(\gamma)]+1} x^{\rho+\alpha+\alpha'-\gamma-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j}$$
$$\times \aleph^{m+3,n}_{p_i+3,q_i+3,\tau_i;r}\left[zx^{\mu} \,\middle|\, \begin{matrix} (a_j,A_j)_{1,n}, (1-\rho-\sum_{j=1}^s \lambda_j k_j,\mu), (1-\alpha'+\beta'-\rho-\sum_{j=1}^s \lambda_j k_j,\mu), \\ (1-\alpha-\alpha'+\gamma-\rho-\sum_{j=1}^s \lambda_j k_j,\mu), (1+\beta'-\rho-\sum_{j=1}^s \lambda_j k_j,\mu), \end{matrix}\right.$$
$$\left. \begin{matrix} (1-\alpha-\alpha'-\beta+\gamma-\rho-\sum_{j=1}^s \lambda_j k_j,\mu), \ldots, [\tau_j(a_j,A_j)]_{n+1,p_i} \\ (1-\alpha'-\beta+\gamma-\rho-\sum_{j=1}^s \lambda_j k_j,\mu), (b_j,B_j), \ldots, [\tau_j(b_j,B_j)]_{m+1,q_i} \end{matrix} \right]. \tag{3.4}$$

Proof In order to prove (3.4), first expressing the general class of multivariable polynomials occurring on its left-hand side as a series given by (1.25), replacing the $\aleph$-function in terms of Mellin-Barnes contour integral with the help of (1.17), and interchanging the order of summations, we obtain the following form (say I):

$$I = \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!}$$
$$\times \left\{ \frac{1}{2\pi i} \int_L \Omega^{m,n}_{p_i,q_i,\tau_i;r} z^{-\xi} \left(D_-^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho+\sum_{j=1}^s \lambda_j k_j-\mu\xi-1}\right)(x)\, d\xi \right\}$$
$$= \left(-\frac{d}{dx}\right)^k \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!}$$
$$\times \frac{1}{2\pi i} \int_L x^{\rho+\alpha+\alpha'-\gamma+k+\sum_{j=1}^s \lambda_j k_j-1} \left(zx^{\mu}\right)^{-\xi}$$
$$\times \frac{\prod_{j=1}^m \Gamma(b_j+B_j\xi) \prod_{j=1}^n \Gamma(1-a_j-A_j\xi)}{\sum_{i=1}^r \tau_i \prod_{j=m+1}^{q_i} \Gamma(1-b_{ji}-B_{ji}\xi) \prod_{j=n+1}^{p_i} \Gamma(a_{ji}+A_{ji}\xi)}$$
$$\times \frac{\Gamma(1-\alpha-\alpha'+\gamma-k-\rho-\sum_{j=1}^s \lambda_j k_j+\mu\xi)}{\Gamma(1-\rho-\sum_{j=1}^s \lambda_j k_j+\mu\xi)}$$
$$\times \frac{\Gamma(1-\alpha'-\beta+\gamma-\rho-\sum_{j=1}^s \lambda_j k_j+\mu\xi)\Gamma(1+\beta'-\rho-\sum_{j=1}^s \lambda_j k_j+\mu\xi)}{\Gamma(1-\alpha-\alpha'-\beta+\gamma-\rho-\sum_{j=1}^s \lambda_j k_j+\mu\xi)\Gamma(1-\alpha'+\beta'-\rho-\sum_{j=1}^s \lambda_j k_j+\mu\xi)}\, d\xi.$$

Here $k := [\Re(\gamma)]+1$, and by using (3.2) in the above expression, and re-interpreting the Mellin-Barnes contour integral in terms of the $\aleph$-function, we are led to the right-hand side of (3.4). This completes the proof of Theorem 4. □

In view of the relation (1.14), we obtain a (presumably) new result concerning Saigo fractional derivative operator [16] asserted by the following corollary.

Corollary 4 *Let* $\alpha,\beta,\gamma,\rho,z \in \mathbb{C}$, $\Re(\alpha)>0$, $\mu>0$, $\lambda_j \in \Re_+$ $(j=1,\ldots,s)$, *and*

$$\Re(\rho) + \mu \max_{1\leq i\leq n}\left(\frac{\Re(a_i)-1}{A_i}\right) < 1 + \min\left\{0, \left[\Re(\alpha)\right]-\Re(\beta)-1, \Re(\alpha+\gamma)\right\}.$$

Then the following relation holds true:

$$\left\{ D_-^{\alpha,\beta,\gamma} \left(t^{\rho-1} S_L^{h_1,\ldots,h_s}\left(y_1 t^{\lambda_1},\ldots,y_s t^{\lambda_s}\right) \right.\right.$$
$$\left.\left. \times \aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[zt^{\mu} \,\middle|\, \begin{matrix} (a_j,A_j)_{1,n}, \ldots, [\tau_j(a_j,A_j)]_{n+1,p_i} \\ (b_j,B_j)_{1,m}, \ldots, [\tau_j(b_j,B_j)]_{m+1,q_i} \end{matrix} \right] \right) \right\}(x)$$

$$= (-1)^{[\Re(\alpha)]+1} x^{\rho+\beta-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j}$$
$$\times \aleph^{m+2,n}_{p_i+2,q_i+2,\tau_i;r}\left[zx^\mu \,\middle|\, \begin{matrix} (a_j,A_j)_{1,n}, (1-\rho-\sum_{j=1}^s \lambda_jk_j,\mu), \\ (1-\beta-\rho-\sum_{j=1}^s\lambda_jk_j,\mu), (1+\alpha+\gamma-\rho-\sum_{j=1}^s\lambda_jk_j,\mu), \end{matrix}\right.$$
$$\left.\begin{matrix} (1-\beta+\gamma-\rho-\sum_{j=1}^s\lambda_jk_j,\mu),\ldots,[\tau_j(a_j,A_j)]_{n+1,p_i} \\ (b_j,B_j),\ldots,[\tau_j(b_j,B_j)]_{m+1,q_i} \end{matrix}\right], \tag{3.5}$$

where the conditions of existence of (3.5) *follow easily from Theorem* 4.

It is remarked in passing that the corresponding results concerning Riemann-Liouville and Erdélyi-Kober fractional derivative operators can be obtained by putting $\beta = -\alpha$ and $\beta = 0$, respectively, in (3.5).

4 Special cases and applications

Here we consider further interesting special cases of Theorem 1. Similarly we can present certain interesting special cases of Theorems 2-4, which are omitted.

(i) If we put $\tau_i = 1$, $i = \overline{1,r}$ in Theorem 1 and take (1.23) into account, then the Aleph function reduces to the I-function as follows (see [29]):

$$\Bigg\{ I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\Bigg(t^{\rho-1} S_L^{h_1,\ldots,h_s}(y_1t^{\lambda_1},\ldots,y_st^{\lambda_s})$$
$$\times I^{m,n}_{p_i,q_i;r}\left[zt^\mu \,\middle|\, \begin{matrix}(a_j,A_j)_{1,n},\ldots,(a_j,A_j)_{n+1,p_i} \\ (b_j,B_j)_{1,m},\ldots,(b_j,B_j)_{m+1,q_i}\end{matrix}\right]\Bigg)\Bigg\}(x)$$
$$= x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j}$$
$$\times I^{m,n+3}_{p_i+3,q_i+3;r}\left[zx^\mu \,\middle|\, \begin{matrix}(1-\rho-\sum_{j=1}^s\lambda_jk_j,\mu), (1-\rho+\alpha+\alpha'+\beta-\gamma-\sum_{j=1}^s\lambda_jk_j,\mu), \\ (b_j,B_j)_{1,m}, (1-\rho+\alpha+\alpha'-\gamma-\sum_{j=1}^s\lambda_jk_j,\mu),\end{matrix}\right.$$
$$\left.\begin{matrix}(1-\rho+\alpha'-\beta'-\sum_{j=1}^s\lambda_jk_j,\mu), (a_j,A_j)_{1,n},\ldots,(a_j,A_j)_{n+1,p_i} \\ (1-\rho+\alpha'+\beta-\gamma-\sum_{j=1}^s\lambda_jk_j,\mu), (1-\rho-\beta'-\sum_{j=1}^s\lambda_jk_j,\mu),\ldots,(b_j,B_j)_{m+1,q_i}\end{matrix}\right]. \tag{4.1}$$

(ii) If we put $\tau_i = 1$, $i = \overline{1,r}$ and set $r = 1$ in Theorem 1 and take (1.24) into account, then the Aleph function reduces to the H-function as follows (see [30]):

$$\Bigg\{ I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\Bigg(t^{\rho-1} S_L^{h_1,\ldots,h_s}(y_1t^{\lambda_1},\ldots,y_st^{\lambda_s}) H^{m,n}_{p,q}\left[zt^\mu \,\middle|\, \begin{matrix}(a_p,A_p) \\ (b_q,B_q)\end{matrix}\right]\Bigg)\Bigg\}(x)$$
$$= x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j}$$
$$\times H^{m,n+3}_{p+3,q+3}\left[zx^\mu \,\middle|\, \begin{matrix}(1-\rho-\sum_{j=1}^s\lambda_jk_j,\mu), (1-\rho+\alpha+\alpha'+\beta-\gamma-\sum_{j=1}^s\lambda_jk_j,\mu), \\ (b_q,B_q), (1-\rho+\alpha+\alpha'-\gamma-\sum_{j=1}^s\lambda_jk_j,\mu),\end{matrix}\right.$$
$$\left.\begin{matrix}(1-\rho+\alpha'-\beta'-\sum_{j=1}^s\lambda_jk_j,\mu), (a_p,A_p) \\ (1-\rho+\alpha'+\beta-\gamma-\sum_{j=1}^s\lambda_jk_j,\mu), (1-\rho-\beta'-\sum_{j=1}^s\lambda_jk_j,\mu)\end{matrix}\right]. \tag{4.2}$$

(iii) If we use a known relation between the Mittag-Leffler function $E_{\beta,\gamma}^{\delta}$ and the H-function (see Mathai *et al.* [30, p.25, Eq. (1.137)]):

$$E_{\beta,\gamma}^{\delta}(z) = \frac{1}{\Gamma(\delta)} H_{1,2}^{1,1}\left[-z \,\middle|\, \begin{matrix} (0,1),(1-\gamma,\beta) \\ (1-\delta,1) \end{matrix}\right], \tag{4.3}$$

in (4.2), we obtain the following interesting formula:

$$\begin{aligned} &\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1} S_L^{h_1,\ldots,h_s}\left(y_1 t^{\lambda_1},\ldots,y_s t^{\lambda_s}\right) E_{\upsilon,\eta}^{\delta}\left[z t^{\mu}\right]\right)\right\}(x) \\ &\quad = x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j} \\ &\qquad \times \frac{1}{\Gamma(\delta)} H_{4,5}^{1,4}\left[-zx^{\mu} \,\middle|\, \begin{matrix} (1-\rho-\sum_{j=1}^s \lambda_j k_j,\mu),(1-\rho+\alpha+\alpha'+\beta-\gamma-\sum_{j=1}^s \lambda_j k_j,\mu), \\ (0,1),(1-\eta,\upsilon),(1-\rho+\alpha+\alpha'-\gamma-\sum_{j=1}^s \lambda_j k_j,\mu), \end{matrix}\right. \\ &\qquad\qquad \left. \begin{matrix} (1-\rho+\alpha'-\beta'-\sum_{j=1}^s \lambda_j k_j,\mu),(1-\delta,1) \\ (1-\rho+\alpha'+\beta-\gamma-\sum_{j=1}^s \lambda_j k_j,\mu),(1-\rho-\beta'-\sum_{j=1}^s \lambda_j k_j,\mu) \end{matrix}\right]. \end{aligned} \tag{4.4}$$

(iv) If we use a known relation involving the generalized Wright hypergeometric function ${}_p\psi_q$ (see [30, p.25, Eq. (1.140)] in (4.2), we obtain the following interesting formula:

$$\begin{aligned} &\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1} S_L^{h_1,\ldots,h_s}\left(y_1 t^{\lambda_1},\ldots,y_s t^{\lambda_s}\right){}_p\psi_q\left[z t^{\mu} \,\middle|\, \begin{matrix} (a_p,A_p) \\ (b_q,B_q) \end{matrix}\right]\right)\right\}(x) \\ &\quad = x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j} \\ &\qquad \times H_{p+3,q+4}^{1,p+3}\left[-zx^{\mu} \,\middle|\, \begin{matrix} (1-\rho-\sum_{j=1}^s \lambda_j k_j,\mu),(1-\rho+\alpha+\alpha'+\beta-\gamma-\sum_{j=1}^s \lambda_j k_j,\mu), \\ (0,1),(1-b_q,B_q),(1-\rho+\alpha+\alpha'-\gamma-\sum_{j=1}^s \lambda_j k_j,\mu), \end{matrix}\right. \\ &\qquad\qquad \left. \begin{matrix} (1-\rho+\alpha'-\beta'-\sum_{j=1}^s \lambda_j k_j,\mu),(1-a_p,A_p) \\ (1-\rho+\alpha'+\beta-\gamma-\sum_{j=1}^s \lambda_j k_j,\mu),(1-\rho-\beta'-\sum_{j=1}^s \lambda_j k_j,\mu) \end{matrix}\right]. \end{aligned} \tag{4.5}$$

(v) If we use the following known relation between the H-function and the generalized Bessel-Maitland function $J_{\nu,\lambda}^{\mu}(z)$ (see Mathai *et al.* [30, p.25, Eq. (1.139)]):

$$H_{1,3}^{1,1}\left[\frac{z^2}{4} \,\middle|\, \begin{matrix} (\lambda+\nu/2,1) \\ (\lambda+\nu/2,1),(\nu/2,1),(\mu(\lambda+\nu/2)-\lambda-\nu,\mu) \end{matrix}\right] = J_{\nu,\lambda}^{\mu}(z) \tag{4.6}$$

in (4.2), we get the following interesting formula:

$$\begin{aligned} &\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1} S_L^{h_1,\ldots,h_s}\left(y_1 t^{\lambda_1},\ldots,y_s t^{\lambda_s}\right) J_{\nu,\lambda}^{\eta}\left(z t^{\mu}\right)\right)\right\}(x) \\ &\quad = x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\leq L} (-L)_{h_1k_1+\cdots+h_sk_s} A(L;k_1,\ldots,k_s) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{\sum_{j=1}^s \lambda_j k_j} \\ &\qquad \times H_{4,6}^{1,4}\left[\frac{(zx^{\mu})^2}{4} \,\middle|\, \begin{matrix} (1-\rho-\sum_{j=1}^s \lambda_j k_j,\mu),(1-\rho+\alpha+\alpha'+\beta-\gamma-\sum_{j=1}^s \lambda_j k_j,\mu), \\ (\lambda+\nu/2,1),(\nu/2,1),(\eta(\lambda+\nu/2)-\lambda-\nu,\eta),(1-\rho-\beta'-\sum_{j=1}^s \lambda_j k_j,\mu), \end{matrix}\right. \end{aligned}$$

$$\left.\begin{matrix} (1-\rho+\alpha'-\beta'-\sum_{j=1}^{s}\lambda_j k_j,\mu),(\lambda+\nu/2,1) \\ (1-\rho+\alpha+\alpha'-\gamma-\sum_{j=1}^{s}\lambda_j k_j,\mu),(1-\rho+\alpha'+\beta-\gamma-\sum_{j=1}^{s}\lambda_j k_j,\mu) \end{matrix}\right]. \tag{4.7}$$

It is noted that many other relations involving some known special functions can be obtained as special cases of (4.2).

(vi) If we set a general class of multivariable polynomials $S_L^{h_1,\dots,h_s}$ to unity, then we easily get the results given by Ram and Kumar [19].

(vii) Further, if we set a general class of multivariable polynomials $S_L^{h_1,\dots,h_s}$ to unity, and reduce the $\aleph$-function to Fox's H-function, then we can easily obtain the known results given by Saxena and Saigo [23].

5 Conclusion

In the present paper, we have given the four theorems of generalized fractional integral and derivative operators given by Saigo-Maeda. The theorems have been developed in terms of the product of $\aleph$-function and a general class of multivariable polynomials in a compact and elegant form with the help of Saigo-Maeda power function formulas. Most of the given results have been put in a compact form, avoiding the occurrence of infinite series and thus making them useful in applications.

In view of the generality of the $\aleph$-function, on specializing the various parameters, we can obtain from our results, several results involving a remarkably wide variety of useful functions, which are expressible in terms of the H-function, the I-function, the G-function of one variable and their various special cases. Secondly, on suitably specializing the various parameters of the general class of multivariable variable polynomials, our results can be reduced to a large number of fractional calculus results involving the general class of polynomials, Jacobi polynomials, Legendre polynomials, Hermite polynomials, Bessel polynomials, Gould-Hopper polynomials, and their various particular cases. Thus, the results presented in this paper would at once yield a very large number of results involving a large variety of special functions occurring in the problems of science, engineering, and mathematical physics *etc.*

Competing interests
The authors declare that there is no conflict of interests regarding the publication of this article.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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